3 optimization

Take it to the max

We all want more of something.

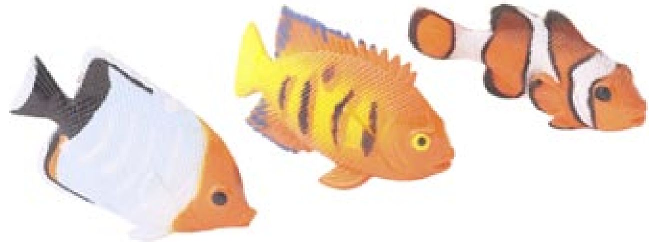
And we're always trying to figure out how to get it. *If* the things we want more of—profit, money, efficiency, speed—can be represented numerically, then chances are, there's an tool of data analysis to help us tweak our *decision variables,* which will help us find the **solution** or *optimal point* where we get the most of what we want. In this chapter, you'll be using one of those tools and the powerful spreadsheet **Solver** package that implements it.

You're now in the bath toy game

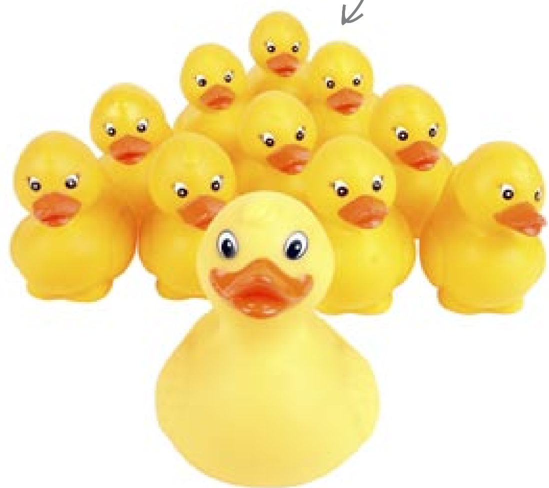
You've been hired by Bathing Friends Unlimited, one of the country's premier manufactures of rubber duckies and fish for bath-time entertainment purposes. Believe it or not, bath toys are a serious and profitable business.

They want to make more money, and they hear that managing their business through data analysis is all the rage, so they called you!

The rubber fish is an unconventional choice, but it's been a big seller.



Some call it the classic, some say it's too obvious, but one thing is clear: the rubber ducky is here to stay.



I'll give your firm top consideration as I make my toy purchases this year.

Duckies make me giggle.

You have demanding, discerning customers.

Sharpen your pencil

Here's an email from your client at Bathing Friends Unlimited, describing why they hired you.

From: Bathing Friends Unlimited
To: Head First
Subject: Requested analysis of product mix

Dear Analyst,

We're excited to have you!

We want to be as profitable as possible, and in order to get our profits up, we need to make sure we're making the right amount of ducks and the right amount of fish. What we need you to help us figure out is our ideal *product mix*: how much of each should we manufacture?

Looking forward to your work. We've heard great things.

Regards,

BFU

Here's what your client says about what she needs.

What ***data*** do you need to solve this problem?

....

....

....

....

....

....

Sharpen your pencil Solution

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To: Head First
Subject: Requested analysis of product mix

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Regards,

BFU

What ***data*** do you need to solve this problem?

First of all, it'd be nice to have data on just how profitable ducks and fish are. Is one more profitable than the other? But more than that, it'd be nice to know what other factors constrain the problem. How much rubber does it take make these products? And how much time does it take to manufacture these products?

Your Data Needs Up Close

Take a closer look at what you need to know. You can divide those data needs into two categories: **things you can't control**, and things you can.

These are things you can't control.

- How profitable fish are
- How much rubber they have to make fish
- How much rubber they have to make ducks
- How profitable ducks are
- How much time it takes to make fish
- How much time it takes to make ducks

And the basic thing the client wants you to find out in order to get the profit as high as possible. Ultimately, the answers to these two questions you **can control**.

These are things you can control.

- How many fish to make
- How many ducks to make

You need the hard numbers on what you can and can't control.

Constraints limit the variables you control

These considerations are called **constraints**, because they will define the parameters for your problem. What you're ultimately after is *profit*, and finding the right product mix is how you'll determine the right level of profitability for next month.

But your options for product mix will be ***limited*** by your constraints.

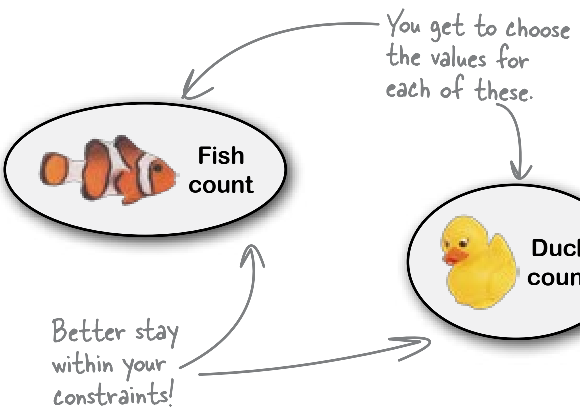
These are your actual constraints for this problem.

> **From: Bathing Friends Unlimited**
> **To: Head First**
> **Subject: Potentially useful info**
>
> **Dear Analyst,**
>
> **Great questions. Re rubber supply: we have enough rubber to manufacture 500 ducks or 400 fish. If we did make 400 fish, we wouldn't have any rubber to make ducks, and vice versa.**
>
> **We have time to make 400 ducks or 300 fish. That has to do with the time it takes to set the rubber. No matter what the product mix is, we can't make more than 400 ducks and 300 fish if we want the product on shelves next month.**
>
> **Finally, each duck makes us $5 in profit, and each fish makes us $4 in profit. Does that help?**
>
> **Regards,**
>
> **BFU**

Decision variables are things you can control

Constraints don't tell you how to maximize profit; they only tell you what you *can't* do to maximize profit.

Decision variables, on the other hand, are the things you ***can*** control. You get to choose how many ducks and fish will be manufactured, and as long as your constraints are met, your job is to choose the combination that creates the most profit.



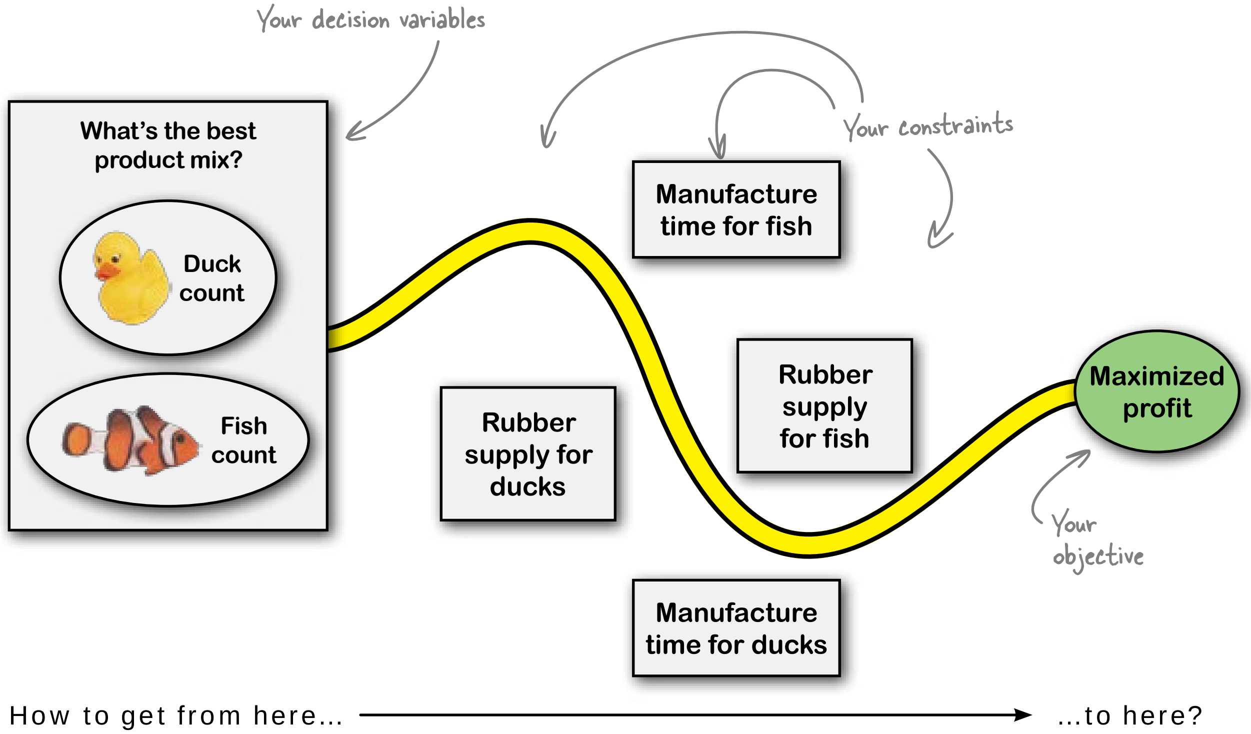
BRAIN POWER

So, what do you think you ***do*** with constraints and decision variables to figure out how to maximize profit?

You have an optimization problem

When you want to get as much (or as little) of something as possible, and the way you'll get it is by changing the values of other quantities, you have an **optimization problem**.

Here you want to maximize *profit* by changing your decision variables: the number of ducks and fish you manufacture.



But to maximize profit, you have to stay within your constraints: the manufacture time and rubber supply for both toys.

To solve an optimization problem, you need to combine your decision variables, constraints, and the thing you want to maximize together into an **objective function**.

Find your objective with the objective function

The **objective** is the thing you want to maximize or minimize, and you use the **objective function** to find the optimum result.

Here's what your objective function looks like, if you state it algebraically:

Each "c" refers to a constraint.

Each "x" refers to a decision variable.

"P" is your objective: the thing you want to maximize.

$$c_1x_1 + c_2x_2 = P$$

Some optimization problems have more elaborate objective functions.

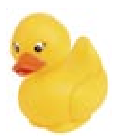
Don't be scared! All this equation says is that you should get the highest P (profit) possible by multiplying each decision variable by a constraint.

Your constraints and decision variables in this equation combine to become the profit of ducks and fish, and those together form your objective: the total profit.

And here's c_2x_2

You want your objective to be as high as you can get it.

Here's c_1x_1



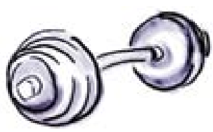
duck profit

\+

fish profit

= Total Profit

All optimization problems have constraints and an objective function.

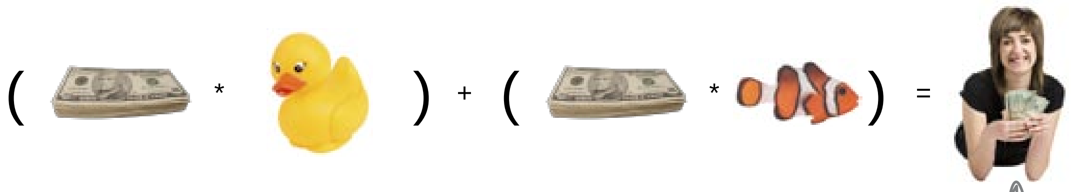


BRAIN BARBELL

What specific values do you think you should use for the constraints, c_1 and c_2?

Your objective function

The constraints that you need to put into your objective function are the **profit for each toy**. Here's another way to look at that algebraic function:



Here's your client from Bathing Friends Unlimited.

The profit you get from selling fish and ducks is equal to the profit per duck multiplied by the number of ducks plus the profit per fish multiplied by the number of fish.

$$\underbrace{(\text{profit per duck} * \text{count of ducks})}_{\text{Total duck profit.}} + \underbrace{(\text{profit per fish} * \text{count of fish})}_{\text{Total fish profit.}} = \text{Profit}$$

Now you can start trying out some product mixes. You can fill in this equation with the values you know represent the profit per item along with some hypothetical count amounts.

This is what your profit would be if you decide to make 100 ducks and 50 fish.

$$(\$5\text{ profit} * 100\text{ ducks}) + (\$4\text{ profit} * 50\text{ fish}) = \$700$$

This objective function projects a $700 profit for *next month*. We'll use the objective function to try out a number of other product mixes, too.

Hey! What about all those other constraints? Like rubber and time?

Show product mixes with your other constraints

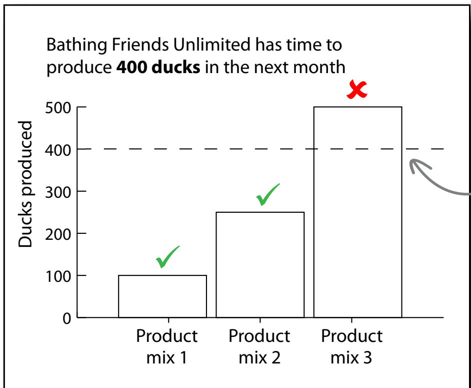
Rubber and time place limits on the count of fish you can manufacture, and the best way to start thinking about these constraints is to envision different hypothetical **product mixes**. Let's start with the constraint of *time*.

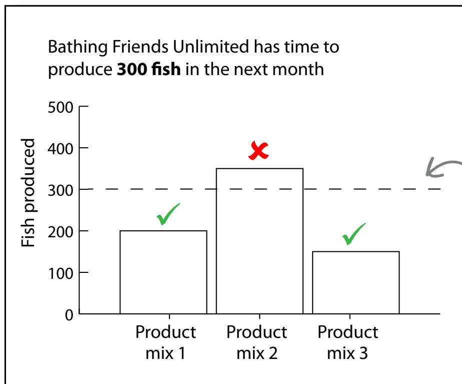
Here's what they say about their time constraint.

> ducks, and vice versa.
>
> **We have time to make 400 ducks or 300 fish. That has to do with the time it takes to set the rubber. No matter what the product mix is, we can't make more than 400 ducks and 300 fish if we want the product on shelves next month.**
>
> **Finally, each duck makes us $5 in profit,**

A hypothetical "Product mix 1" might be where you manufacture 100 ducks and 200 fish. You can plot the time constraints for that product mix (and two others) on these bar graphs.

This line shows the maximum number of ducks you can produce.

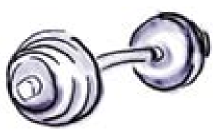




This line shows how many fish you have time to produce.

Product mix 1 doesn't violate any constraints, but the other two do: product mix 2 has too many fish, and product mix 3 has too many ducks.

Seeing the constraints in this way is progress, but we need a better visualization. We have yet more constraints to manage, and it'd be clearer if we could view them **both** on a single chart.

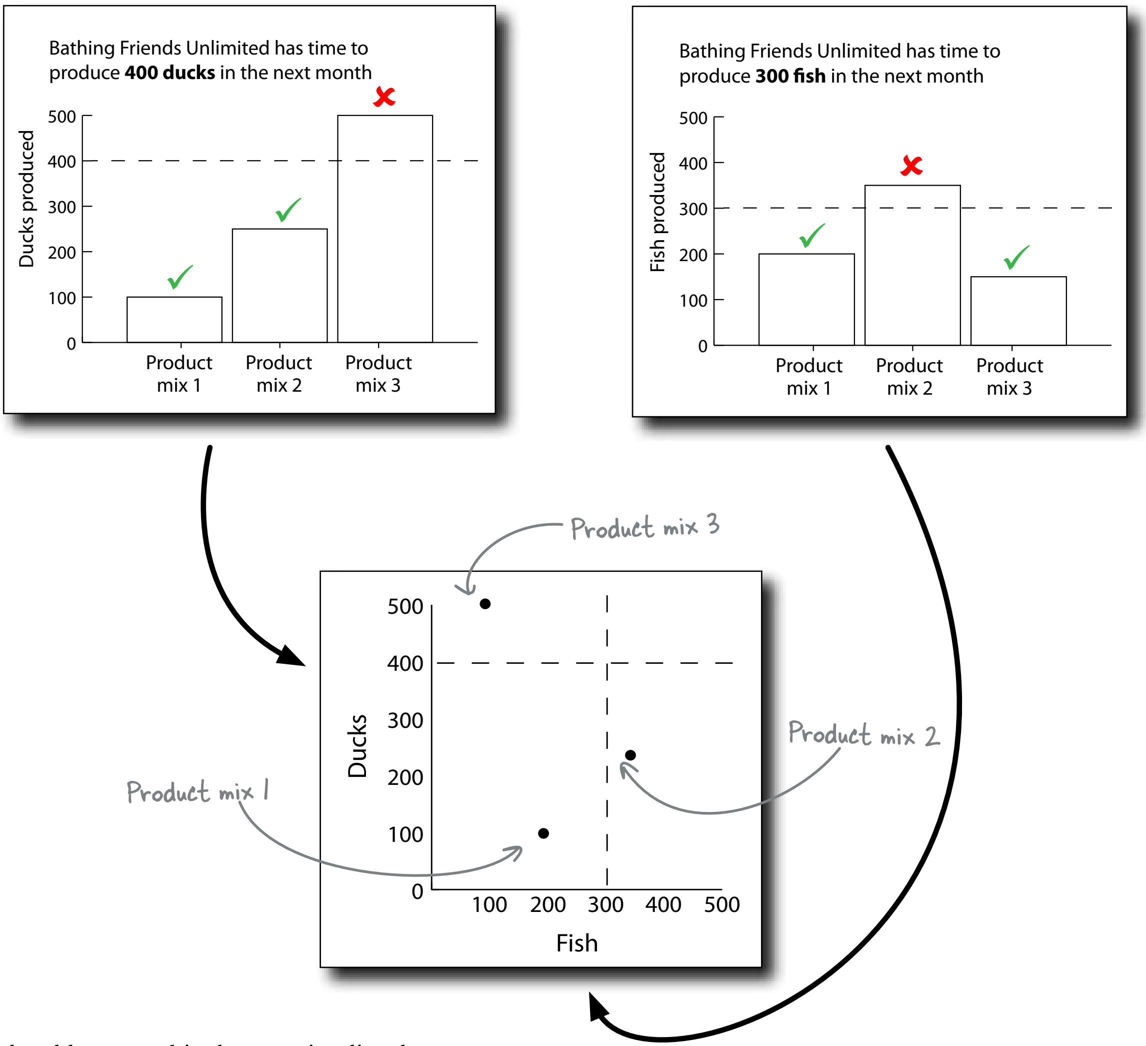


BRAIN BARBELL

How would you visualize the constraints on hypothetical product mixes of ducks *and* fish with one chart?

Plot multiple constraints on the same chart

We can plot both time constraints on a single chart, representing each product mix with a dot rather than a bar. The resulting chart makes it easy to **visualize both time constraints together**.



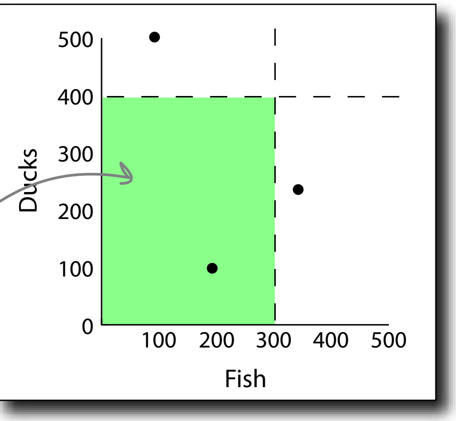
We'll also be able to use this chart to visualize the rubber constraints. In fact, you can place **any number of constraints** on this chart and get an idea of what product mixes are possible.

Your good options are all in the feasible region

Plotting ducks on a y-axis and fish on an x-axis makes it easy to see what product mixes are *feasible*. In fact, the space where product mixes are within the constraint lines is called the **feasible region**.

When you add constraints to your chart, the feasible region will change, and you'll use the feasible region to figure out which point is *optimal*.

This is the feasible region.



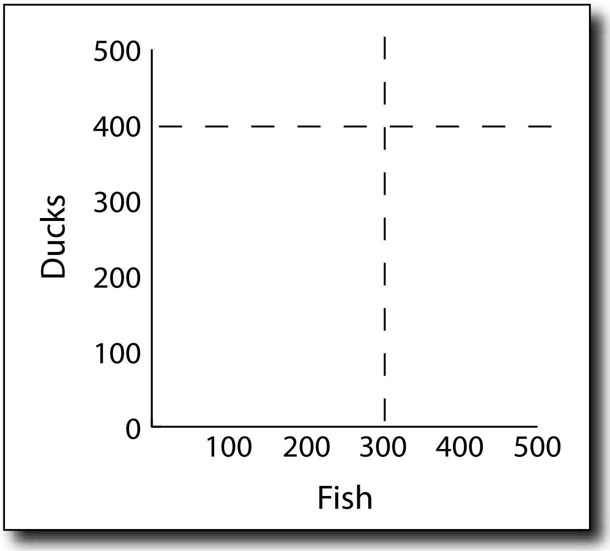
Sharpen your pencil

Let's add our other constraint, which states how many fish and ducks can be produced given the quantity of rubber they have. Bathing Friends Unlimited said:

Each fish takes a little more rubber to make than each duck.

Great questions. Re rubber supply: we have enough rubber to manufacture 500 ducks or 400 fish. If we did make 400 fish, we wouldn't have any rubber to make ducks, and vice versa.

You have a fixed supply of rubber, so the number of ducks you make will limit the number of fish you can make.



1. Draw a point representing a product mix where you make 400 fish. As she says, if you make 400 fish, you won't have rubber to make any ducks.

2. Draw a point representing a product mix where you make 500 ducks. If you made 500 ducks, you'd be able to make zero fish.

3. Draw a line through the two points.

Sharpen your pencil Solution

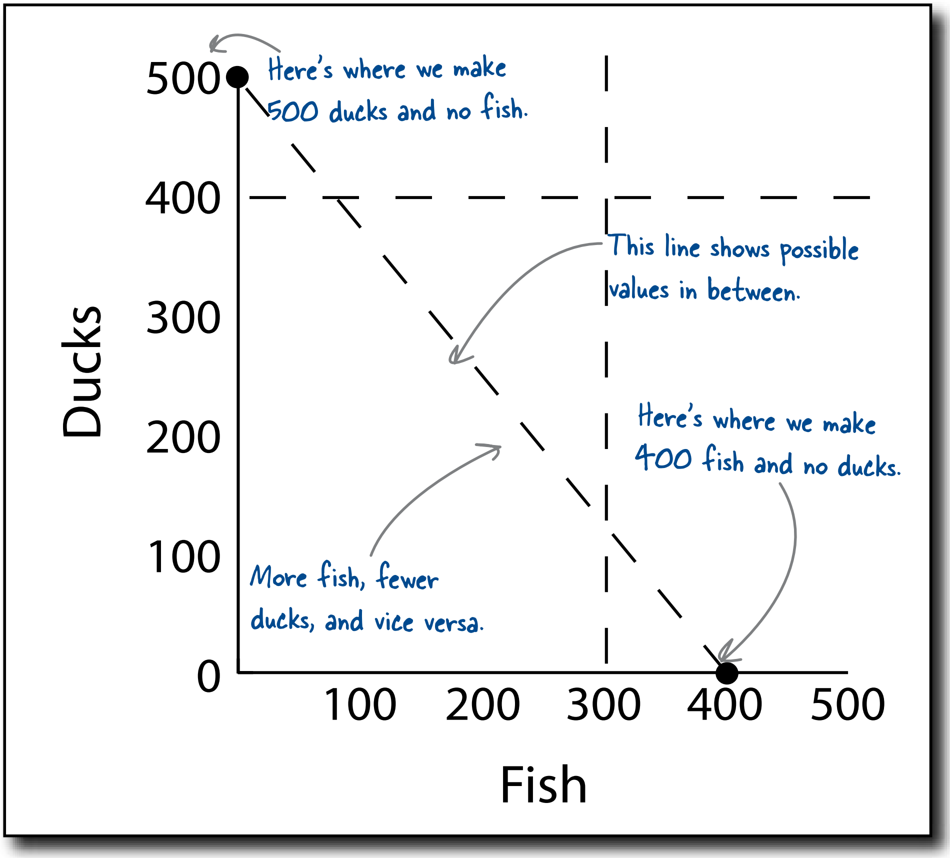
How does the new constraint look on your chart?

1. Draw a point representing a product mix where you make 400 fish. As she says, if you make 400 fish, you won't have rubber to make any ducks.

2. Draw a point representing a product mix where you make 500 ducks. If you made 500 ducks, you'd be able to make zero fish.

3. Draw a line through the two points.

Great questions. Re rubber supply: we have enough rubber to manufacture 500 ducks or 400 fish. If we did make 400 fish, we wouldn't have any rubber to make ducks, and vice versa.

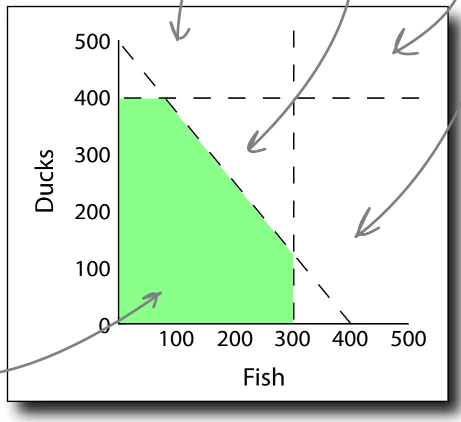


Your new constraint changed the feasible region

When you added the rubber constraint, you **changed the shape** of the feasible region.

Before you added the constraint, you might have been able to make, say, 400 ducks and 300 fish. But now your rubber scarcity has ruled out that product mix as a possibility.

You can't use duck/fish combinations that exist in any of these spaces.



Your potential product mixes all need to be inside here.

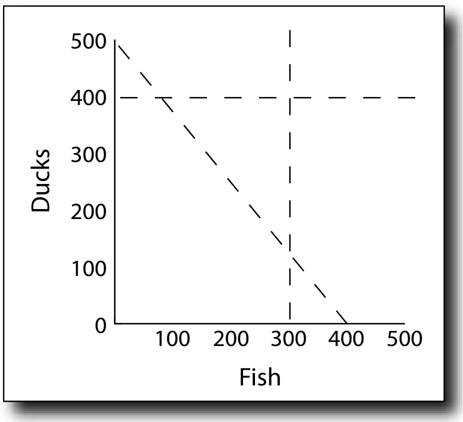
Sharpen your pencil

Here are some possible product mixes.

Are they inside the feasible region?
Draw a dot for each product mix on the chart.

How much profit will the different product mixes create?
Use the equation below to determine the profit for each.

Draw where each product mix goes on the chart.



300 ducks and 250 fish

Profit: ....

100 ducks and 200 fish

Profit: ....

50 ducks and 300 fish

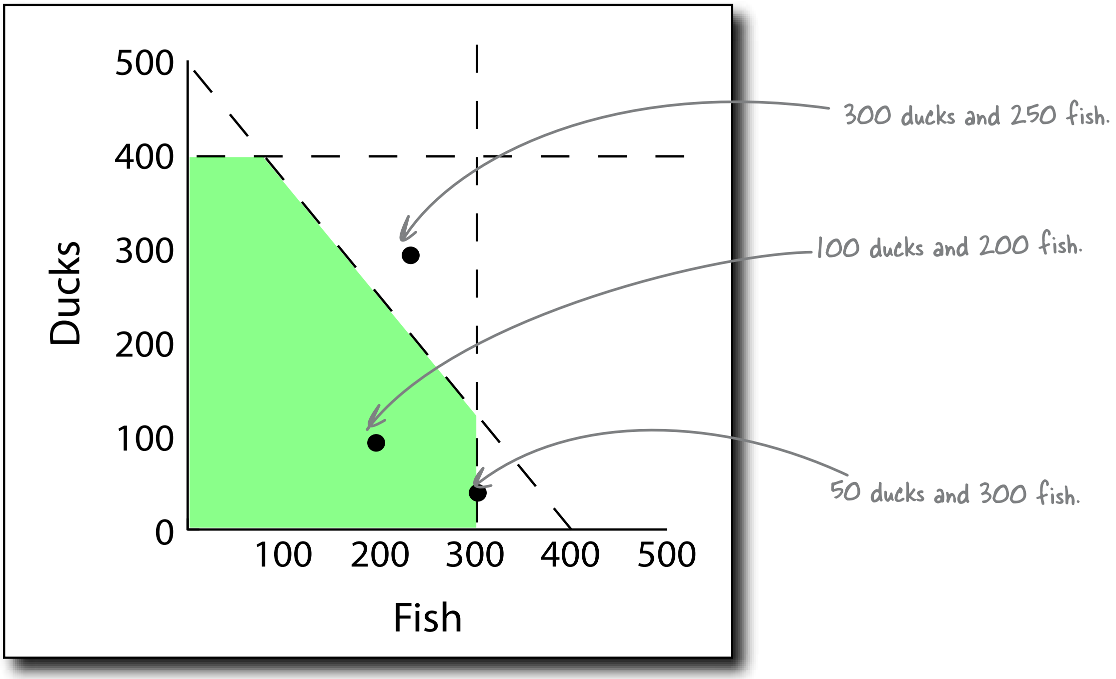
Profit: ....

Use your objective function to determine profit.

(\$5 profit * count of ducks) + (\$4 profit * count of fish) = Profit

Sharpen your pencil Solution

You just graphed and calculated the profit for three different product mixes of ducks and fish. What did you find?



300 ducks and 250 fish.

Profit: ($5 profit*300 ducks)+($4 profit*250 fish) = $2500

Too bad this product mix isn't in the feasible region.

100 ducks and 200 fish.

Profit: ($5 profit*100 ducks)+($4 profit*200 fish) = $1300

This product mix definitely works.

50 ducks and 300 fish.

Profit: ($5 profit*50 ducks)+($4 profit*300 fish) = $1450

This product mix works and makes even more money.

Now all you have to do is try every possible product mix and see which one has the most profit, right?

You don't have to try them all.

Because both Microsoft Excel and OpenOffice have a handy little function that makes short order of optimization problems. Just turn the page to find out how…

Your spreadsheet does optimization

Microsoft Excel and OpenOffice both have a handy little utility called **Solver** that can make short order of your optimization problems.

If you plug in the constraints and write the objective function, Solver does the algebra for you. Take a look at this spreadsheet, which describes all the information you received from Bathing Friends Unlimited.

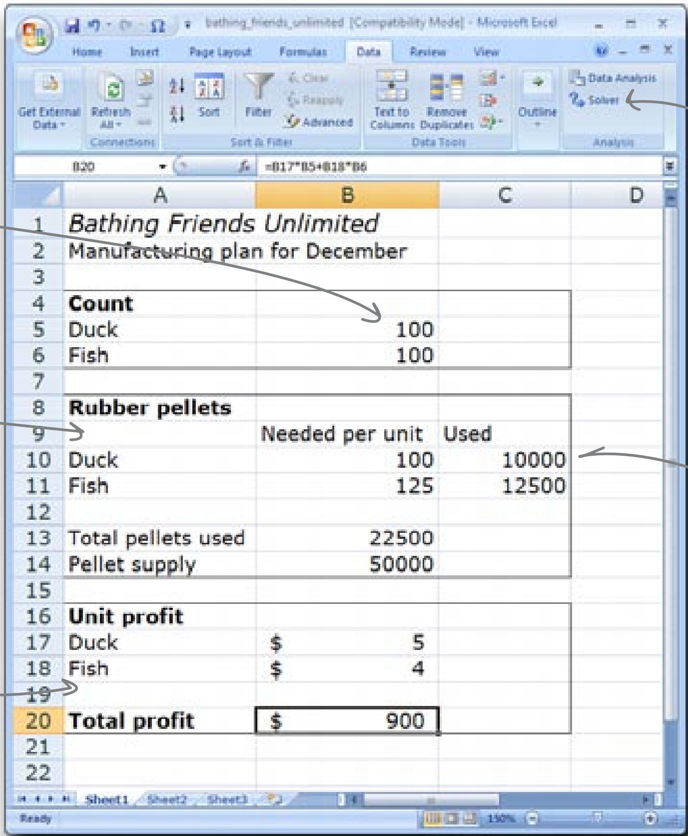
Load this!

www.headfirstlabs.com/books/hfda/bathing_friends_unlimited.xls

These cells show a product mix where you manufacture 100 ducks and fish each.

This box shows your rubber supply.

This box shows your profit.



	A	B	C
1	*Bathing Friends Unlimited*		
2	Manufacturing plan for December		
3			
4	**Count**		
5	Duck	100	
6	Fish	100	
7			
8	**Rubber pellets**		
9		Needed per unit	Used
10	Duck	100	10000
11	Fish	125	12500
12			
13	Total pellets used	22500	
14	Pellet supply	50000	
15			
16	**Unit profit**		
17	Duck	$ 5	
18	Fish	$ 4	
19			
20	**Total profit**	$ 900	
21			
22			

Here's Solver.

If you make 100 ducks at 100 rubber pellets per duck, you use 100,000 pellets.

There are a few simple formulas on this spreadsheet. First, here are some numbers to quantify your rubber needs. The bath toys are made out of rubber pellets, and cells `B10:B11` have formulas that calculate how many pellets you need.

Second, cell `B20` has a formula that multiplies the count of fish and ducks by the profit for each to get the total profit.

Take a look at Appendix iii if you use OpenOffice or if Solver isn't on your Excel menu.

Try clicking the Solver button under the Data tab. What happens?

Sharpen your pencil

Let's take a look at the Solver dialogue box and figure out how it works with the concepts you've learned.

Draw an arrow from each element to where it goes in the Solver dialogue box.

Rubber and time

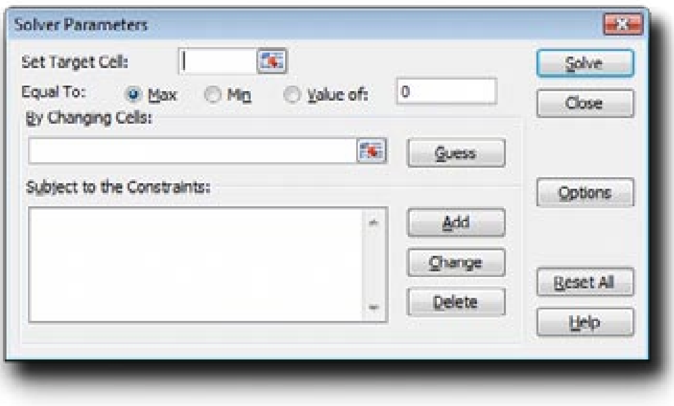
Decision variables

Constraints

Objective

The number of ducks to make

Profit



Draw an arrow from each element to where it should go on the Solver.

Where do you think the **objective function** goes?

....

....

Sharpen your pencil Solution

How do the spaces in the Solver dialogue box match up with the optimization concepts you've learned?

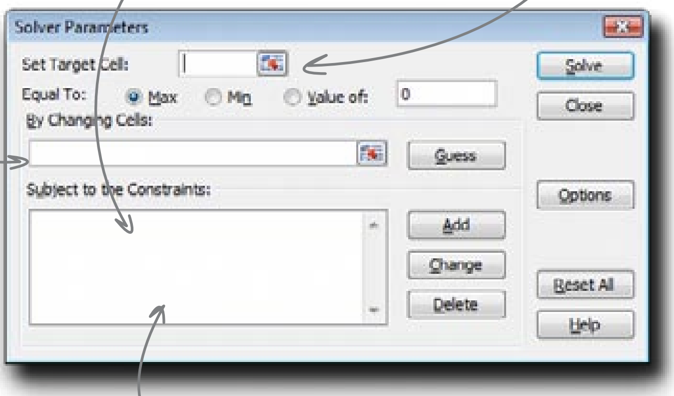
Draw an arrow from each element to where it goes in the Solver dialogue box.

Decision variables

Constraints

Objective

Excel calls your objective the Target Cell.



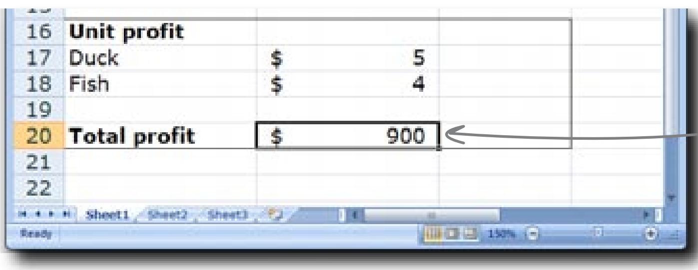
The decision variables are the values you will change to find your objective.

Constraints go in the constraints box... no big surprise there!

Where do you think the **objective function** goes?

The objective function goes in a cell on the spreadsheet and returns the objective as the result.

The objective that this objective function calculates is the total profit.



The objective function is in this cell.

Test Drive

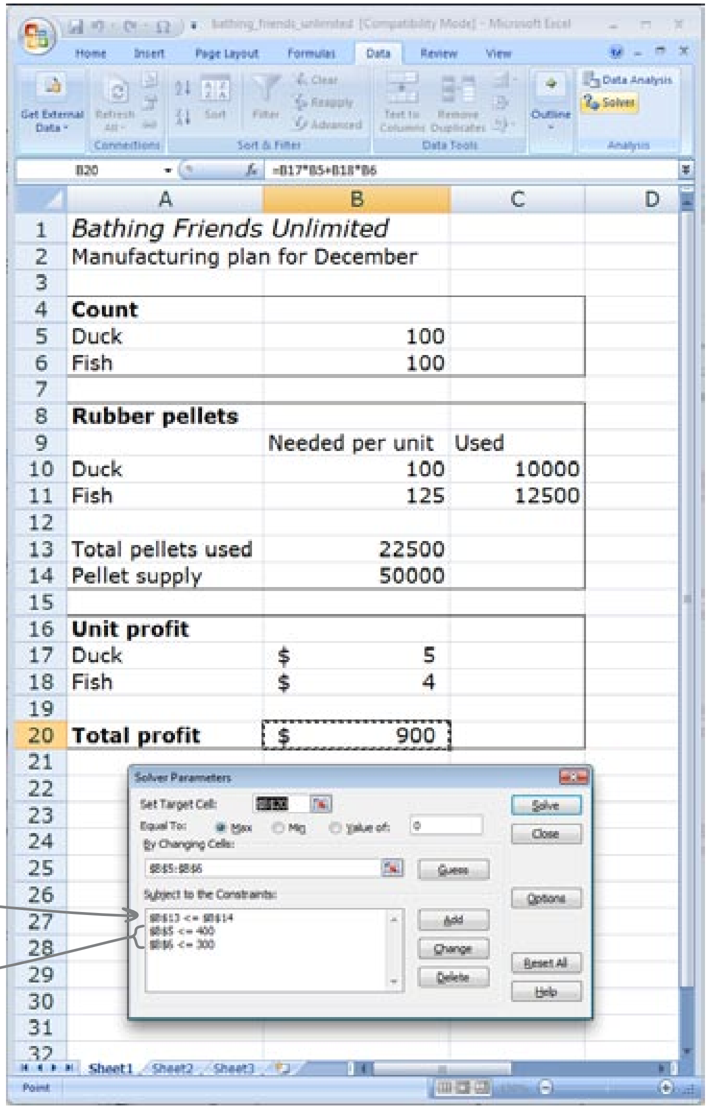
Now that you've defined your optimization model, it's time to plug the elements of it into Excel and let the Solver do your number crunching for you.

1. Set your target cell to point to your objective function.

2. Find your decision variables and add them to the Changing Cells blank.

3. Add your constraints.

4. Click Solve!



Here's your rubber constraint.

Don't forget your time constraints!

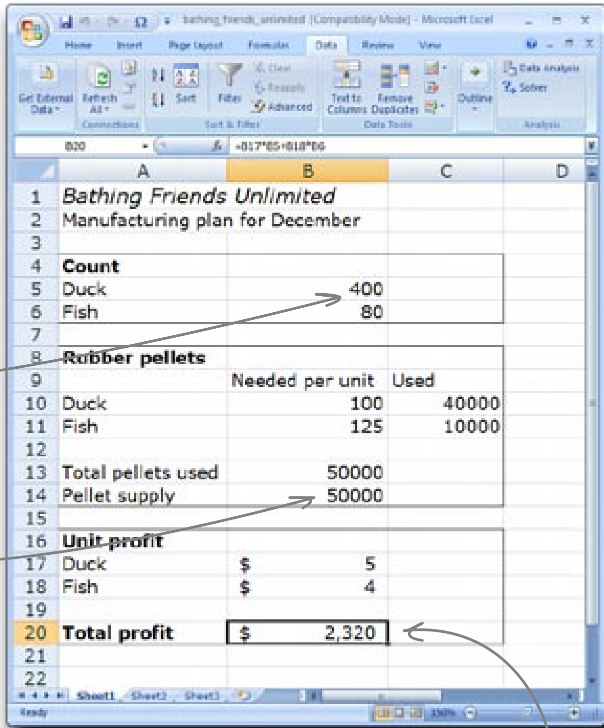
What happens when you click Solve?

Solver crunched your optimization problem in a snap

Nice work. Solver took all of about a millisecond to find the solution to your optimization problem. If Bathing Friends Unlimited wants to maximize its profit, it need only manufacture 400 ducks and 80 fish.

Solver tried out a bunch of Count values and found the ones that maximize profit.

Looks like you're using all your rubber, too.



	A	B	C
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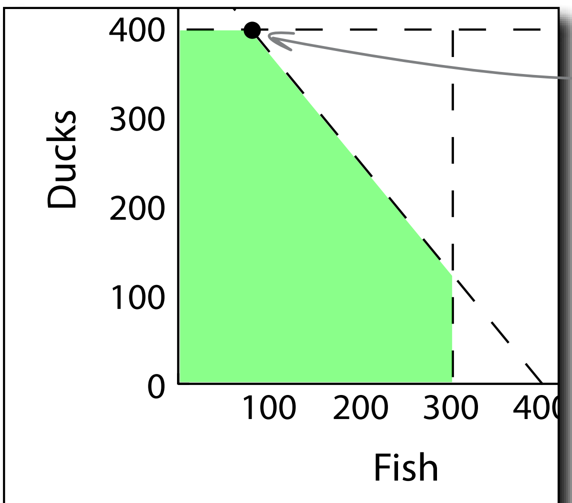
2	Manufacturing plan for December		
3			
4	**Count**		
5	Duck	400	
6	Fish	80	
7			
8	**Rubber pellets**		
9		Needed per unit	Used
10	Duck	100	40000
11	Fish	125	10000
12			
13	Total pellets used	50000	
14	Pellet supply	50000	
15			
16	**Unit profit**		
17	Duck	$ 5	
18	Fish	$ 4	
19			
20	**Total profit**	$ 2,320	
21			
22			

What's more, if you compare Solver's result to the graph you created, you can see that the precise point that Solver considers the best is on the outer limit of your feasible region.



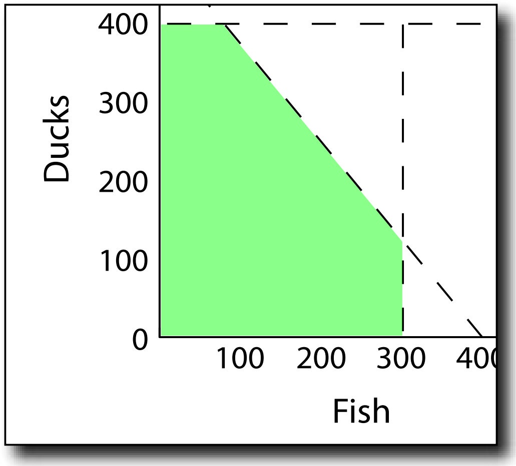
Here's your solution.

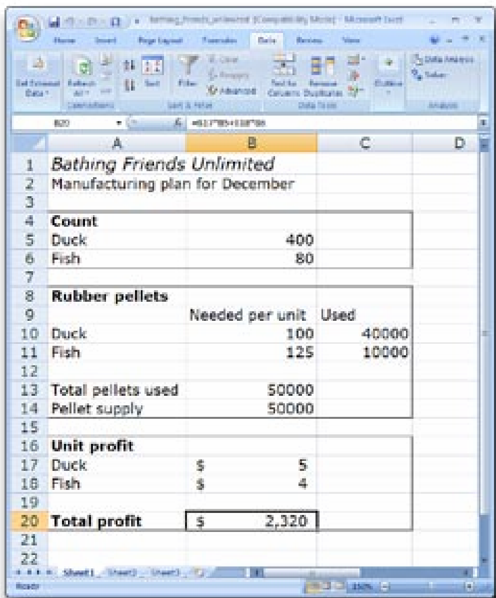
Here's the profit you can expect.

Better explain to the client what you've been up to...

Sharpen your pencil

How would you explain to the client what you're up to? Describe each of these visualizations. What do they mean, and what do they accomplish?





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19			
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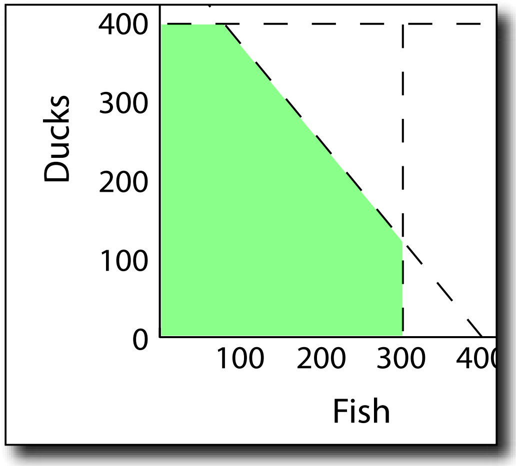
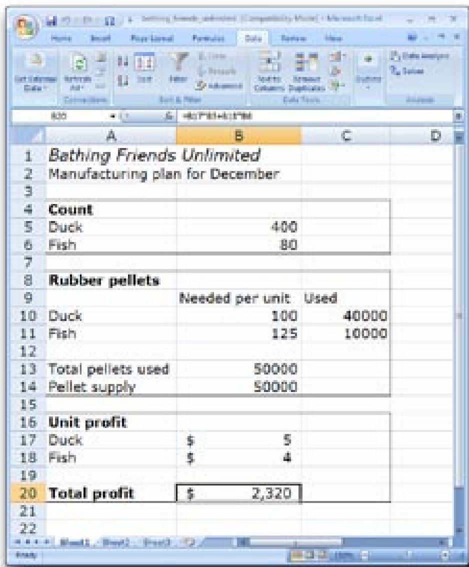
21			
22			

Sharpen your pencil Solution

How did you interpret your findings to your client?



The shaded part of this graph shows all the possible duck/fish product mixes given our constraints, which are represented by the dashed lines. But this chart does not point out the solution itself.



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15			
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17	Duck	$ 5	
18	Fish	$ 4	
19			
20	**Total profit**	$ 2,320	
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22			

This spreadsheet shows the product mix computed by Excel to be the optimum. Of all possible product mixes, manufacturing 400 ducks and 80 fish produces the most profit while staying inside our constraints.